Equivalents & Reciprocals

$$\square \div \square$$

Fractions can be interpreted as the division of a numerator by a denominator...
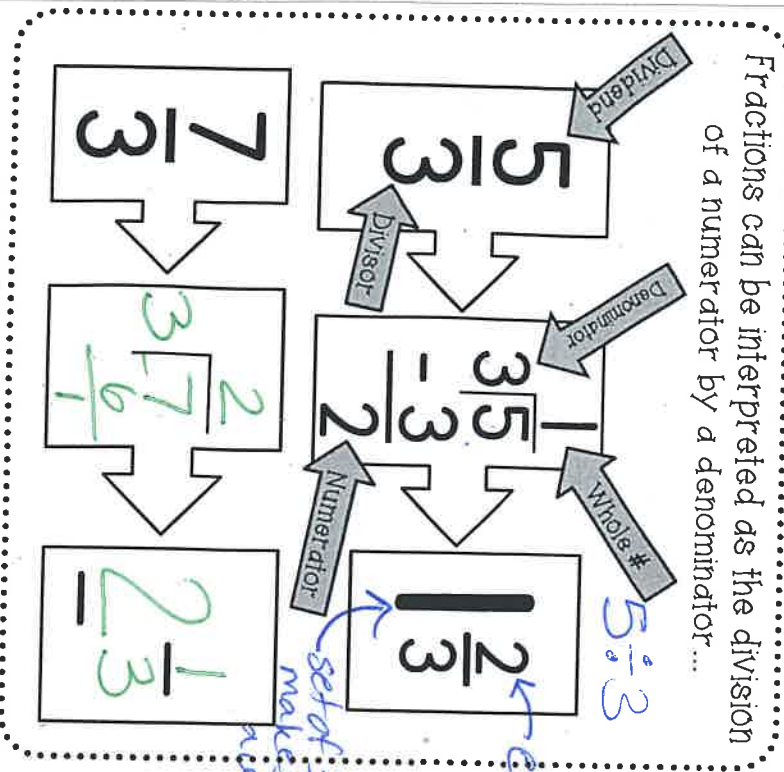
⓪

$\dfrac{5}{3}$ ← Dividend

$\dfrac{3\overline{)5}}{-3}$ ← Divisor, Denominator

$\underline{-3}$
2 ← Whole # $5 \div 3$

$1\dfrac{2}{3}$ ← extra pieces (N)   ← set of 3 makes a whole

①

$\dfrac{7}{3} \rightarrow \dfrac{2}{3\overline{)7}}\;\dfrac{-6}{1} \rightarrow 2\dfrac{1}{3}$

②

$\dfrac{6}{5} \rightarrow 5\overline{)6} \rightarrow 1\dfrac{1}{5}$

③

$\dfrac{8}{3} \rightarrow \boxed{} \rightarrow 1\dfrac{1}{}$

$\dfrac{27}{5}\;\dfrac{5}{5}\overline{)27}\;\dfrac{25}{2}\;5\dfrac{2}{5}$

④

$4\dfrac{2}{5}$

$4\overset{+}{\underset{\times}{\dfrac{2}{5}}} = \dfrac{2}{5}$ $4 \times 5 + 2$

⑤

$5\overset{+}{\underset{\times}{\dfrac{1}{3}}} = \dfrac{16}{3}$ ← total pieces

$3 \times 5 + 1$

⑥

$3\overset{+}{\underset{\times}{\dfrac{8}{9}}} = \dfrac{9 \times 3 + 8}{9}$ $\dfrac{27 + 8}{}$ $\dfrac{35}{9}$

⑦

$1\dfrac{2}{} =$

Reciprocal: The reciprocal of a fraction is the fraction turned upside-down.

$\frac{8}{1} \rightarrow$ 8 ⤸ 8 ⤸ 1/8

$\frac{2}{3}$ ⤸ 3/2

The reciprocal is not equal.
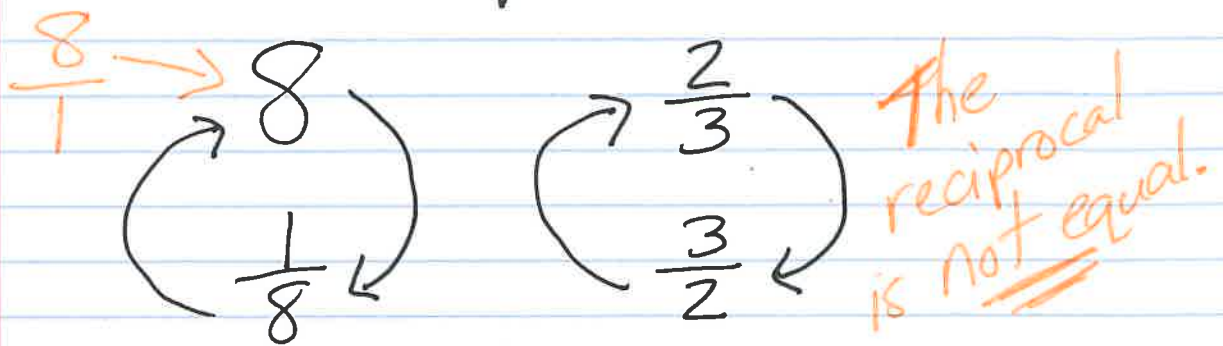
$\frac{4}{5}$ reciprocal $\frac{5}{4}$

$\frac{3}{9}$ $\frac{9}{3}$

$\frac{11}{5}$ $\frac{5}{11}$ $\frac{13}{5}$ $\frac{5}{13}$

$5 \times 2 + 3$

$\frac{7}{10}$ $\frac{10}{7}$

DIVIDING FRACTIONS

Remember!

Keep	Change	Flip
First fraction stays the same	Operation changes from ÷ to ×	Flip 2ⁿ fraction for reciprocal

	Fraction divided by a fraction	Whole number divided by a fraction	Fraction divided by a mixed number
Step 1: Write whole number as fraction; write mixed number as improper fraction.	$\frac{2}{3} \div \frac{1}{3}$	$9 \div \frac{1}{3}$ $\downarrow$ $\frac{9}{1} \div \frac{1}{3}$	$\frac{2}{3} \div 2\frac{1}{3}$ $\downarrow$ $\frac{2}{3} \div \frac{7}{3}$
Step 2: Find the reciprocal of the divisor (the number you are dividing by).	$\frac{2}{3} \div \frac{3}{1}$	$\frac{9}{1} \div \frac{3}{1}$	$\frac{2}{3} \div \frac{3}{7}$
Step 3: The reciprocal allows you to change the operation from division to multiplication.	$\frac{2}{3} \times \frac{3}{1}$	$\frac{9}{1} \times \frac{3}{1}$	$\frac{2}{3} \times \frac{3}{7}$
Step 4: Multiply the fractions.	$\frac{2}{3} \times \frac{3}{1} = \frac{6}{3}$	$\frac{9}{1} \times \frac{3}{1} = \frac{27}{1}$	$\frac{2}{3} \times \frac{3}{7} = \frac{6}{21}$
Step 5: Write the answer in simplest terms.	$\frac{6}{3} = 2$	$\frac{27}{1} = 27$	$\frac{6}{21} = \frac{2}{7}$

Handwritten (top margin):
$$\frac{1}{9} \div \frac{3}{4} = \frac{7}{9} \times \frac{4}{3} = \frac{28}{27} = 1\frac{1}{27}$$

Handwritten (left margin): $\frac{1}{4} \div \frac{2}{8}$

LESSON 4.2 Dividing Fractions

ESSENTIAL QUESTION
How do you divide fractions?

COMMON CORE 6.NS.1
Interpret and compute quotients of fractions,..., e.g. by using visual fraction models,....

EXPLORE ACTIVITY 1 — Real World

COMMON CORE 6.NS.1

Modeling Fraction Division

In some division problems, you may know a number of groups and need to find how many or how much are in each group. In other division problems, you may know how many there are in each group, and need to find the number of groups.

A You have $\frac{3}{4}$ cup of salsa for making burritos. Each burrito requires $\frac{1}{8}$ cup of salsa. How many burritos can you make? ___ **6 burritos**

To find the number of burritos that can be made, you need to determine how many $\frac{1}{8}$-cup servings are in $\frac{3}{4}$ cups. Use the diagram. How many eighths are there in $\frac{3}{4}$? ___

You have enough salsa to make ___ burritos.

Handwritten: $\frac{3}{4} \div \frac{1}{8}$ $\frac{3}{4} \times \frac{8}{1} = \frac{24}{4} = 6$ $4\overline{)24}$

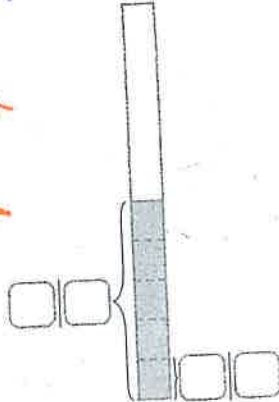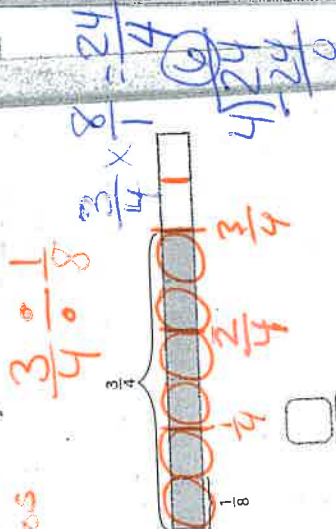**B** Five people share $\frac{1}{2}$ pound of cheese equally. How much cheese does each person receive?

To find how much cheese each person receives, you need to determine how much is in each of ___ parts.

How much is in each part? ___

Each person will receive ___ pound.

Reflect
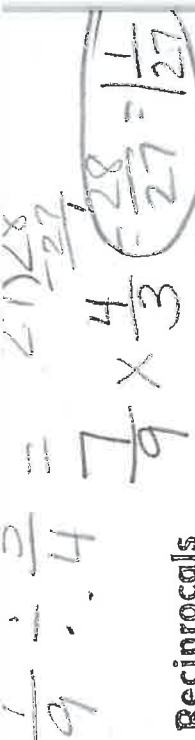1. Write the division shown by each model.

Lesson 4.2   85

Reciprocals

Another way to divide fractions is to use *reciprocals*. Two numbers whose product is 1 are *reciprocals*.

$\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$ $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals.

To find the reciprocal of a fraction, switch the numerator and denominator.

$\frac{\text{numerator}}{\text{denominator}} \cdot \frac{\text{denominator}}{\text{numerator}} = 1$

COMMON CORE   Prep for 6.NS.1

EXAMPLE 1

Find the reciprocal of each number.

A $\frac{2}{9}$ Switch the numerator and denominator.

The reciprocal of $\frac{2}{9}$ is $\frac{9}{2}$.

B $\frac{1}{8}$ Switch the numerator and denominator.

The reciprocal of $\frac{1}{8}$ is $\frac{8}{1}$, or 8.

C 5   Rewrite as a fraction.

$5 = \frac{5}{1}$

$\frac{5}{1}$ → $\frac{1}{5}$ Switch the numerator and the denominator.

The reciprocal of 5 is $\frac{1}{5}$.

Math Talk Mathematical Practices
How can you check that the reciprocal in **A** is correct?

Reflect
2. Is any number its own reciprocal? If so, what number(s)? Justify your answer. **No**

3. **Communicate Mathematical Ideas** Does every number have a reciprocal? Explain.

4. The reciprocal of a whole number is a fraction with ___ in the numerator.

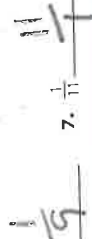### YOUR TURN
Find the reciprocal of each number.

5. $\frac{7}{8}$ $\frac{8}{7}$

6. $\frac{9}{5}$ $\frac{5}{9}$

7. $1\frac{1}{11}$ $\frac{11}{1}$

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86   Unit 2

Dividing Fractions
Keep, Change, Flip!

1. **Keep** the first fraction.
2. **Change** the sign to multiplication.
3. **Flip** the second fraction into its reciprocal.
4. **Multiply** the problem through $= \dfrac{\text{numerator x numerator}}{\text{denominator x denominator}}$
5. **Simplify.** (Use the ladder method if you're stuck!)

Example: $\dfrac{2}{8} \div \dfrac{3}{4}$

1. **Keep** $\dfrac{2}{8}$
2. **Change** $\div$ to x
3. **Flip** $\dfrac{3}{4}$ to $\dfrac{4}{3}$
4. **Multiply** $\dfrac{2}{8} x \dfrac{4}{3} = \dfrac{8}{24}$
5. **Simplify** $\dfrac{8}{24} = \dfrac{1}{3}$

2	8 / 24
2	4 / 12
2	2 / 6
2	1 / 3	→ Simplified fraction

have/has ÷ size = groups
have/has ÷ group = size per

(margin, right, blue) dividing, splitting, distributing, sharing EQUALLY

Name _____ Class _____ Date _____

4.2 Independent Practice

COMMON CORE 6.NS.1

Starts with

8. Alison has ½ cup of yogurt for making fruit parfaits. Each parfait requires ⅛ cup of yogurt. How many parfaits can she make?

___4 parfaits___

9. A team of runners is needed to run a ¼-mile relay race. If each runner must run $\frac{1}{16}$ mile, how many runners will be needed?

(margin: groups, ?)

10. Trevor paints ⅙ of the fence surrounding his farm each day. How many days will it take him to paint ¾ of the fence?

11. *Have* Six people share ⅖ pound of peanuts equally. What fraction of a pound of peanuts does each person receive?

$\frac{2}{5} \div 6$ *Size?*

(margin left: size of each group, groups)

12. **Biology** If one honeybee makes $\frac{1}{12}$ teaspoon of honey during its lifetime, how many honeybees are needed to make ½ teaspoon of honey?
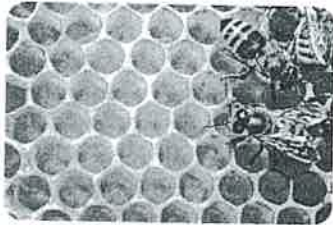
13. Jackson wants to divide a ¾-pound box of trail mix into small bags. Each of the bags will hold $\frac{1}{12}$ pound of trail mix. How many bags of trail mix can Jackson fill?

___9 bags___   *size*   *have*

14. A pitcher of contains ⅔ quart of lemonade. If an equal amount of lemonade is poured into each of 6 glasses, how much lemonade will each glass contain?

$\frac{2}{3} \div 6$

(margin: HAVE groups, size?)

15. How many tenths are there in ⅘ ?

16. You make a large bowl of salad to share with your friends. Your brother eats ⅓ of it before they come over.

a. You want to divide the leftover salad evenly among six friends. What expression describes the situation? Explain.

_____
_____
_____
_____

b. What fractional portion of the original bowl of salad does each friend receive?

_____
_____
_____

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Lesson 4.2 **89**

⑧ $\frac{1}{2} \div \frac{1}{8}$

$\frac{1}{2} \times \frac{8}{1} = \frac{8}{2} = 4$ parfaits

$2\overline{)8} = 4$, -8, 0

⑬ $\frac{3}{4} \div \frac{1}{12} =$

$\frac{3}{4} \times \frac{12}{1} = \frac{36}{4}$

$4\overline{)36} = 9$, -36, 0

⑨ bags

Dividing w/ Mixed Numbers

$$5 \frac{2}{3} \div \frac{2}{3}$$

How many $\frac{2}{3}$ are in $5 \frac{2}{3}$?

$3 \times 5 + 2$

$$\frac{17}{3} \div \frac{2}{3} = \frac{17}{3} \times \frac{3}{2} = \frac{51}{6}$$

$$6 \overline{)51} \quad \frac{\overset{8}{}}{} $$

$$6 \overline{)51} \qquad 8 \frac{3 \div 3}{6 \div 3} = \boxed{8 \frac{1}{2}}$$
$$\underline{-48}$$
$$3$$

$5 \frac{2}{6} = 8 \frac{3}{6} = 8 \frac{1}{2}$

$$3 \frac{1}{5} \div \frac{2}{7}$$

$11 \times 10 = 110$

$$\frac{16}{5} \div \frac{2}{7} \qquad \frac{16}{5} \times \frac{7}{2} = \frac{112}{10} \overset{-110}{}$$

$$11 \frac{2}{10} = 11 \frac{2 \div 1}{2 \cdot 5} = \boxed{11 \frac{1}{5}}$$

$$6 \div 2\frac{4}{9} = \underline{2\frac{5}{11}}$$

$$8 \div \frac{1}{5} = \underline{40}$$

$$9 \div \frac{1}{2} = \underline{18}$$

$$\frac{1}{8} \div 3 = \underline{1/24}$$

$$3\frac{3}{5} \div 1\frac{1}{5} = \underline{3}$$

① $\frac{6}{1} \div \frac{22}{9}$

$\frac{6}{1} \times \frac{9}{22} = \frac{54}{22} = 2\frac{10}{22}$

$2\overline{)10 \quad 22}$

$5 / 11$

$2\frac{5}{11}$

② $\frac{8}{1} \times \frac{5}{1} = \frac{40}{1} = 40$

③ $\frac{9}{1} \times \frac{2}{1} = \frac{18}{1} = 18$

④ $\frac{1}{8} \times \frac{1}{3} = \frac{1}{24}$

⑤ $\frac{18}{5} \div \frac{6}{5} = \frac{18}{5} \times \frac{5}{6}$

$\frac{90}{30} = 3$